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Mathematical model of railway plan

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Abstract: The long-term practice of drawing up a railway plan is based on a curve formed by the projection of the center of the rolling stock onto a horizontal plane. At the same time, this curve is considered flat. The railway plan itself consists of straight parts and a curved part connecting these straight parts. The existence of a curved part in the railway plan is natural. The work substantiates that the railway plan in the curved part can be interpreted as a spatial curve and a natural trihedron can be constructed. With the help of this, the Frenier formula is determined. This formula, from the course on differential curve geometry, is proposed to be accepted as a mathematical model of the railway plan. Frenier's formula is a system of differential equations. Moreover, this system has a solution for a given curvature and torsion. Thus, it is possible to find the equation of the curve of the part of the railway plan.
Keywords: railway plan, radius of a curve, curvature of an osculating circle, osculating plane, railway design, torsion, arc length, osculating trihedron

1. Introduction

The railway plan consists of straight line and curved parts [1].

From an economic point of view, the straight part of the road is more profitable. But the terrain, natural barriers, and settlements do not allow it, it will build a road consisting only of straight parts [2]. Therefore, the straight parts of the road are connected using a curve [3].

When designing railways, the railway plan in the curved part is considered as a flat curve [1],[4]. But there are methods for modeling the railway plan, in which the curved part is considered as a spatial curve [5], [6].

In this paper, it is proved that in the curved part, the railway plan is a spatial curve.

The method of linking the railway to the plan, accompanying the natural trihedron, is indicated [8]. This trihedron allows an analog the Frenier formula for the railway plan. Determining the curvature from the torsion of the curve makes it possible to find the equation of the curve having this curvature and torsion [9]. When determining the initial conditions, the requirements for the geometric shape of the curve of the railway plan are taken into account [10], [11], [12].

The Frenier formula is proposed as a mathematical model of the curve of a part of the railway plan.

2. Research methodology

Main elements of a railway plan

The railway plan is the projection of the track axis onto a horizontal plane. The horizontal plane is understood as a plane that is the tangent plane of the globe.

The railway plan consists of straight and curved sections. Straight sections can be part of a straight line lying on a horizontal plane or a plane other than horizontal. Curvilinear sections can also be flat or spatial.

From an operational point of view, it is convenient and profitable to have only straight sections on a horizontal plane. Even straight sections of non-horizontal sections are difficult to operate.

But designing a railway in the form of only straight sections is impossible. This impossibility is due to the terrain, natural features and environmental requirements of the area. The population of the area plays an important role in planning. These obstacles make it necessary to connect straight sections with curved sections. There cannot be corner points in a railway plan. But the angle between straight sections is called the line turning angle and is an important parameter of the railway plan [1].

For economic and technical reasons, it is advantageous to use arcs of one circle on curved sections of the railway plan. The radius of this circle is called the path radius. But the topographical and geological conditions of the terrain require the use of more complex curves other than a circle. Sometimes these curves will not be flat.

The turning angle and radius of the curve are important in railway design. Moreover, the rotation angle can take any value depending on the complexity of the roads. But the radius of the curve cannot take arbitrary values. Because with small values of the radius, it is impossible to organize the movement of the train. But from a construction point of view, it is more expedient to have curves of small radii, which will ensure the shortness of the road.

One of the important factors of a road is its brevity, that is, its geodes city. Therefore, straight sections that are geodetic have priority. But in this case, difficulties also arise related to the technical capabilities of transport traction. When a straight section is located on a non-horizontal plane, these difficulties are associated with the rise of the road relative to the horizontal plane. To get around this difficulty, you sometimes have to use a curved road in the form of a spiral.

When designing a railway, one should take into account factors related to the need to limit the speed of trains, lengthening roads, increasing the wear resistance of rails, increasing costs for the ongoing maintenance of the upper structure of the track, reducing the coefficient of adhesion of locomotive wheels to the rail, many facts related to electrification, and avoiding various obstacles.

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The purpose of this article is to build a mathematical model of the railway track, taking into account the given basic elements of the railway track plan.

Modern methods for designing plan and longitudinal profile of railways

Railway planning has quite deep scientific roots; in addition, it is based on many years and repeated experience of operating railways.

The railway plan, as a projection of the track axis onto a horizontal plane, has the shape of a trace of a point, that is, it is a curved, outlined projection of the track axis onto a horizontal plane. (Figure 1).

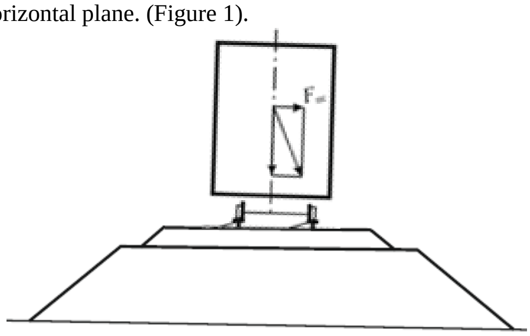

Fig. 1. The railway profile in the straight part of the road

In the straight part of the road, the road plan drawing represents a straight line on a horizontal plane.

But when the train moves along a curved section, the position of the train corresponds to another drawing (Fig. 2).

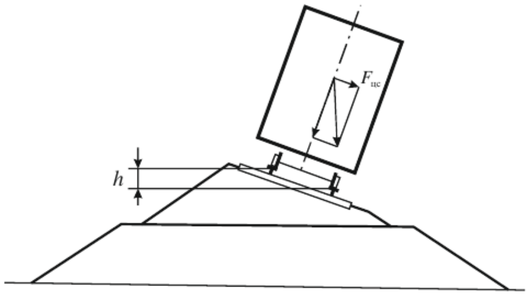

Fig. 2. The railway profile on the curved part of the road

This is explained by the fact that the train is acted upon by a force proportional to the square of the speed of the train and directed from the center of the touching circle of the curve at this point. Under the influence of this force, a significant overload of the outer rail occurs. To avoid this impact, the elevation of the outer rail in relation to the inner one is established [2].

There are regulatory documents that determine the value of the rail elevation so that the overload of the outer rail does not exceed the permissible value. Achieving the required reduction in centrifugal force will be possible by reducing the speed of movement [3].

In addition, there are many normative documents on railway design that take into account the main elements of the railway plan. With such methods, analytical expressions related to the railway plan are used [1]. In many cases, mathematical models are used in the reconstruction of existing railways [4], they are used to reconstruct the plan of railways [5], [6].

E.P. Lenchenkova's candidate dissertation "Development of a mathematical model of the railway route for the reconstruction of the track", using the mathematical model she built, recommendations were given for the reconstruction of existing railways [6].

The idea that the railway route in the curved part will be a spatial curve was expressed in the work of A.S. Panarin "Mathematical models in the routing of railways," but this idea has not been used in practice [7].

In existing scientific works related to the railway track plan model, it is considered as a curve on a horizontal plane. Therefore, the use of these models does not provide the desired accuracy in project implementation.

Railway track plan curve

To clarify the geometry of the curve reflecting the plan of the railway track, we present some theories and curves from differential geometry associated with the spatial curve.

A curve is called plane if all points of the curve belong to the same plane. This plane can be horizontal or arbitrary, that is, spatial relative to the horizontal plane.

For curves, the concepts of curvature and torsion are introduced. The curvature of a curve characterizes its deviation from a straight line, which is tangent to this curve. In order to understand the torsion of a spatial curve, you need to become familiar with the concept of an osculating plane of the curve.

Let γ be some spatial (not lying on the same plane) curve and M a point belonging to this curve. There are infinitely many planes passing through the point M and along the tangent space. Among these planes there is a plane. α −close plane to γ. This plane is called the osculating plane of the curve γat the point M. Obviously, for a plane curve the osculating plane is this plane itself.

If the curve $\gamma$is given by the vector equation

$$\vec{r}(s) = \vec{x}(s)i + \vec{y}(s)j + \vec{z}(s)k \quad (1)$$

where is s −the length of the curve, $\{i, j, k\}$ −the basis vectors and $x(s), y(s), z(s) \in C^2$,

then the touching plane at the points $M(x_0, y_0, z_0)$ is determined by the formula:

$$\begin{vmatrix} x - x_0 & y - y_0 & z - z_0 \\ x'(s_0) & y'(s_0) & z'(s_0) \\ x''(s_0) & y''(s_0) & z''(s_0) \end{vmatrix} = 0. \quad (2)$$

Consider at two points $M(x_0, y_0, z_0)$ and $N(x_0 + \Delta x, y_0 + \Delta y, z_0 + \Delta z)$ an osculating plane of the curve γ. Let's determine the angle $\Delta\psi$ between these planes. Speed change of angle ψ −between touching planes

$$\lim_{\Delta s \to 0} \frac{\Delta\varphi}{\Delta s} = \varphi_s{}' = \sigma \quad (3)$$

is called torsion of a curve γ into points $M(x_0, y_0, z_0)$.

The torsion of plane curves is zero everywhere.

When the curve γdefines the layout of the railway, the tangent curve coincides with the direction (azimuth) of the sections of the railway. For horizontally plane curves, the osculating plane coincides with the horizontal plane. As noted above, this occurs only in the horizontally straight section of the railway track plan.

If we have a vector equation of a curve given by formula (1), then the curvature and torsion of the curve are calculated by the formulas

$$k = |r''(s)| \quad (4)$$

and

$$\sigma = \frac{|(r'r''r''')|}{k^2} \quad (5)$$

The plane perpendicular to the tangent vector of the curve passing through the point M is called the normal plane of the curve. The plane passing through the points of the curve parallel to the tangent vector and perpendicular to the osculating plane of the curve is called the rectifying plane of the curve. The straightening plane coincides with the profile plane of the railway track plan. Each point of the curve is associated with three planes: osculating, normal and rectifiable. These planes are mutually perpendicular and are called the natural trihedron of the curve at the considered point *M*.

When studying the property of a curve in the neighborhood of an arbitrary point M , it turns out to be convenient to choose a Cartesian coordinate system, taking the point *M* of the curve as the origin of the coordinate system, and the axes of the natural trihedron as the coordinate axes.

Let us denote by $\vec{\tau}$ −the unit vector of the tangent curve, $\vec{n}$ −the unit vector directed along the section of the osculating plane and the normal plane, and $\vec{b}$- the unit vector of the osculating plane .

Then, expressing the derivatives $\dot{\tau}, \dot{b}, \dot{n}$along the arc of the curve through these vectors themselves, we obtain Frenier’s formula:

$$\begin{cases} \dot{\tau} = k\vec{n}, \\ \dot{n} = -k\vec{\tau} - \sigma\vec{b}, \\ \dot{b} = \sigma\vec{n}. \end{cases} \qquad (6)$$

Moreover, the coefficients k (4) and σ (5) curvature and torsion of the curve.

In Frenier's formula, and they completely determine the curve. This statement follows from the theorem:

Theorem. Let $k(s)$ and $\sigma(s)$ be any regular functions, and $\sigma(s) > 0$. Then there is a unique curve, accurate to position in space, for which $k(s)$ is the curvature and $\sigma(s)$ is the torsion at the point corresponding to the arc s.

The proof of this theorem can be found in any textbook on differential geometry.

Geometry of the curved part of the route

Curves in the plan of a railway line (Figure 3) are characterized by the following parameters: angle of rotation α, radius, *K* length of the curved part, tangent *T* , bisector *B*, measure *D* and the direction of rotation.

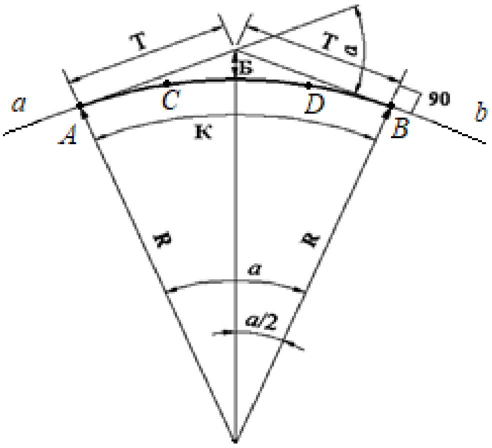

Fig. 3. Curves in the plan of a railway line

Considering that the curved part of the route will be a spatial curve, and this spatial curve should have a beginning at point *A* – the end of the straight part *a*, the end at point *B* the beginning of the straight part *b* , we propose to divide the part of the railway track plan under consideration into three parts. At the same time, arc *AB* – divide into parts *AC*, *CD*, *D.B.*

When entering the curve at the initial point *A*, its junction with the straight section, the rolling stock experiences a side impact, the origin of which is explained by the sudden appearance of centrifugal force. This force acts along the entire length of the curve and disappears at the end point B *as* suddenly as it appeared at the initial point *A.* Such a sudden appearance and disappearance of centrifugal force would disrupt the smooth movement of the train [1].

To ensure a smooth transition from a straight line to a circular curve, and back from the curve to the next straight line, a transition curve of variable curvature is arranged. This is the transition part of the *AC* and *DB* curve. (Figure 3) The quality of the road and maintaining speed significantly depend on the construction of the transition part of the railway plan curve.

In the transition part of the curve, a gradual change in the radius from infinity to a known final value of the curvature of the curve is ensured. In this way, a smooth elevation of the outer rail on the curve is achieved.

The main curved part of the plan is the arc $AC = \gamma_1$ and $DB = \gamma_2$. Since, in order to obtain the desired effect, *h*– the outer part of the rail, is a curved part $\gamma_1$and $\gamma_2$must rise up and become spatial.

The main curved part of the route, its arc *CD*, to have a constant radius R_0. Obviously, the elevation of the outer rail in this part h_0 will also be constant, which is required by the set of rules for designing a railway line plan [7].

The elevation of the outer rail depends on the radius of the curve R_0, and on the square of the train speed in this section. There are various methods and formulas for calculating the elevation of the outer rail. We consider them famous.

The curve is γ_1 to be linearly dependent on *t* the length of the path of his railway plan. Under this condition, it is possible to find the angle of rotation φ when the normal of the horizontal plane is equal to the normal of the contacting plane. Naturally, the angle of rotation φ is a function of *h* the elevation of the outer normal.

It should be noted that part of the curve of the railway plan $CD - b$should actually be flat. But this plane is different from the original horizontal plane.

Consequently, the beginning of the curved part belongs to the horizontal plane, and the end to the spatial plane, that is, to a plane different from the horizontal one.

The listed technical requirements for the curved part of the railway plan make it possible to determine the initial conditions that must be satisfied by the solution of the system of differential equations. [2]

In the simple version under consideration, the curved partγ_1 and γ_2 can be considered symmetrical. Curvature $k(s) -$ can be considered a monotonic continuous function taking the value $\left[-\infty; \frac{1}{R_0}\right]$.

For a spatial curve that has monotonic curvature, the osculating circles do not intersect and lie on different osculating planes.

It is known that the derivative along the length of the arc from the function of the angle between the contacting planes determines the torsion of the curve at the point in question.

Thus, it is possible to determine the torsion of the desired transition part of the curve.

3. Results

In the previous paragraphs, we became familiar with the technical and geometric characteristics of the curved part of the railway plan.

The difficult part of the railway project is its curved part. The problem was considered in the simplest case, when the straight parts of the connecting curve lie on the same horizontal plane. It should be noted that in this simple version, the graph of the transition part of the route curve is represented as a spatial curve. Moreover, for this curve it can determine curvature $k(s)$ as the inverse value of the radius of the curve and torsion $\sigma(s)$ − as the rate of change of the touching plane of the curved route.

From the course of differential geometry, it is known that when the curvature and torsion of a curve are given, it is determined, uniquely up to its position in space, as a solution to the Frenier formula given by formula (6).

Consequently, this system of differential equations can serve as a dynamic system, the transition part of the trace curve.

The conditions listed above, upon request at the beginning of the transition part, serve as the initial conditions of this system of differential equations (6).

The proposed model is a vector system of differential equations.

Taking into account the many years of experience of railway engineers and the simplicity of existing methods, which are repeatedly justified in practice, we can consider a simpler model of a curved route.

Currently, a radial spiral (clothoid) is accepted as a transition curve in the CIS countries, the curvature of which changes in inverse proportion to its current length.

Clotoid or it is called Root spiral, Euler spiral, the equation in parametric form is:

$$\begin{cases} x = \int_0^t cos\left(\frac{\pi u^2}{2}\right) du, \\ y = \int_0^t sin\left(\frac{\pi u^2}{2}\right) du, \end{cases} \quad (7)$$

The integrals involved in the formula are called Fresnel integrals. They are not calculated using elementary methods. The meaning of the parameter t is the length of the Root spiral arc, measured from the origin.

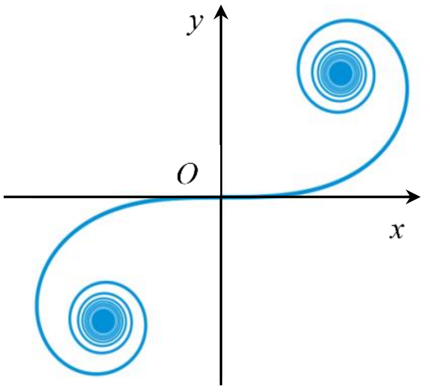

Fig. 4. Greorich clothoids

AC and *DB* considered in the figure is the transition part of the curve. The angle β −is the conditional angle of rotation of this part of the curve.

We denote the curved part of the route corresponding to the incoming part of the curve plan by $\gamma_1$and γ_2- the beginning and end of the curved part, respectively.

To do this, we consider that the plan of the curved part of the railway is defined as a Root spiral given by the parametric equation (7). This means the abscissa and ordinate of the point is determined by system (7). All that remains is to find the applicate (height) $z(s)$ − vector of the function of the defining passage part of the curved route.

Then the system of differential equations (6) is a differential equation for the function $z(s)$.

Having the analytical function of the transition part of the route curve, all the necessary technical characteristics of the path are able to be calculated with the required accuracy.

Therefore, if the curvature and torsion of a curve are known, then it is always possible to determine a curve with these characteristics.

For given terrain parameters, it is always possible to determine the curvature k and torsion σusing the required radii of curvature, and by the value h – the elevation of the outer rail. The railroad plan curve equation can always be calculated as a solution to the system of differential equations (6).

Therefore, we propose a system of differential equations (6) as a mathematical model of the curved part of the railway.

4. Conclusion

When the straight parts of the railway plan are known, that is, the angle between the straight parts and the radius of the assumed curve of the part are given, it is possible to find the curvature of the desired curve, which is the curve of the part of the railway plan.

In addition, if the speed requirement is determined, it is always possible to determine the maximum elevation of the outer rail. This makes it possible to make a requirement for the torsion of the desired curve. When the curvature and torsion of the curve are known and the initial conditions exist, then it is always possible to find the equation of the curve of the railway plan using the Frenier differential equation. Therefore, we propose the Frenier formula as a mathematical model of the railway plan.

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