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Mathematical modeling and artificial intelligence methods for oil filtration processes in multilayer porous media: An analytical review

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Abstract: Oil filtration in porous media is a complex multiphase and nonstationary process that plays an important role in petroleum production, reservoir engineering, and environmental protection. Accurate prediction of filtration behavior requires advanced mathematical models capable of describing fluid transport, phase interactions, and heterogeneity of porous structures. Classical approaches based on Darcy's law, transport equations, multiphase flow models, and computational fluid dynamics (CFD) provide reliable physical descriptions; however, their application is limited by computational complexity and dependence on empirical parameters. This review analyzes modern mathematical and computational approaches for modeling oil filtration in multilayer porous media, including continuum-scale models, pore-scale simulations, digital rock physics, and numerical methods such as finite difference, finite element, and CFD approaches. Particular attention is given to artificial intelligence methods, including machine learning, deep learning, and physics-informed neural networks (PINNs), applied for permeability prediction, pressure field estimation, filtration performance analysis, and process optimization. The analysis shows that hybrid physics–AI approaches provide promising opportunities to overcome the limitations of traditional models by combining physical laws with data-driven learning. Such frameworks can improve prediction accuracy, reduce computational costs, and support the development of intelligent models for nonstationary oil filtration processes in heterogeneous multilayer porous media.

Keywords: oil filtration, porous media, mathematical modeling, Darcy's law, nonstationary flow, multiphase transport, computational fluid dynamics, artificial intelligence, machine learning, physics-informed neural networks

1. Introduction

Oil filtration processes in porous media represent complex transport phenomena that are fundamental for petroleum production, environmental protection, and industrial separation technologies. Efficient separation of oil from water and solid contaminants plays a critical role in produced-water treatment, reservoir management, and improving hydrocarbon recovery efficiency. Fluid transport through porous structures is mainly governed by interactions between pressure gradients, pore geometry, permeability distribution, and multiphase flow mechanisms [1–3]. The mathematical description of oil filtration processes has traditionally been based on continuum transport theories. Darcy's law remains the fundamental equation for describing fluid movement through porous media and has been widely applied in reservoir engineering and filtration analysis [1]. However, real geological formations are characterized by strong heterogeneity, multilayer structures, variable permeability, and nonlinear flow behavior, which significantly complicate accurate prediction of filtration dynamics [2,4].

Classical porous media models describe fluid transport using effective parameters such as permeability, porosity, and relative permeability. The relationship between these parameters is commonly estimated using empirical correlations, including the Kozeny–Carman model [5,6]. Although these approaches provide computational efficiency and physical interpretation, their accuracy decreases for heterogeneous porous systems where pore connectivity, tortuosity, and structural variability cannot be represented by simplified assumptions [3].

In oil-water filtration systems, additional complexity arises from multiphase interactions, including droplet transport, coalescence, deposition, and pore blockage phenomena. Deep-bed filtration theory and colloid transport models have provided important theoretical foundations for describing particle capture mechanisms and filter performance evolution [7,8]. Nevertheless, these models often require experimentally calibrated parameters and may have limited applicability under changing operational conditions.

Recent developments in computational science have enabled more detailed numerical investigation of oil filtration processes. Computational fluid dynamics (CFD), finite element methods (FEM), pore-network modeling, lattice Boltzmann methods, and digital rock physics approaches have significantly improved understanding of fluid transport mechanisms at different spatial scales [9–11]. Digital rock physics, in particular, allows realistic reconstruction of porous structures using micro-computed tomography images and provides opportunities for direct simulation of transport properties [10].

Despite these advances, high computational costs remain a major limitation when applying detailed numerical simulations to large-scale optimization and real-time prediction problems. The increasing availability of experimental data and computational resources has therefore

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accelerated the application of artificial intelligence (AI) and machine learning (ML) methods in porous media modeling.

Machine learning approaches have demonstrated significant potential for predicting permeability, porosity, transport properties, and filtration performance from complex datasets. Deep learning methods, especially convolutional neural networks (CNNs), have enabled automatic extraction of structural features from pore-scale images and improved prediction accuracy compared with conventional empirical correlations [12].

A major breakthrough in scientific machine learning has been the development of physics-informed neural networks (PINNs), which combine artificial intelligence with governing physical equations. PINNs incorporate conservation laws and differential equations directly into neural network training, enabling physically consistent predictions with reduced dependence on large datasets [13]. The method has been successfully applied for solving forward and inverse problems involving nonlinear partial differential equations [13,14].

For porous media and reservoir applications, physics-informed learning provides new opportunities for modeling complex flow and transport processes where experimental data are limited. However, challenges related to nonlinear multiphase transport, model generalization, uncertainty quantification, and computational stability remain important research issues [14,15].

Therefore, this review aims to analyze modern computational modeling and artificial intelligence approaches for nonstationary oil filtration in multilayer porous media. The main objectives are:

- To review mathematical foundations of oil filtration modeling based on porous media transport equations;
- To analyze numerical methods used for continuum and pore-scale simulations;
- To evaluate recent AI and physics-informed machine learning approaches;
- To identify future research directions for developing hybrid AI–physics models for intelligent oil filtration systems.

2. Research methodology

Mathematical Foundations of Oil Filtration Modeling

Fluid flow through porous media is traditionally described by Darcy's law, which establishes the fundamental relationship between pressure gradient, permeability, fluid viscosity, and flow velocity. This law provides the theoretical basis for modeling oil migration and filtration processes in reservoir rocks and engineered filtration systems. In heterogeneous multilayer porous media, the permeability distribution is not constant but varies spatially due to differences in pore structure, geological properties, and fluid–rock interactions. Therefore, advanced models often incorporate spatially variable and pressure-dependent permeability to describe nonstationary filtration behavior more accurately.`

Oil filtration processes in porous media commonly involve simultaneous transport of oil and water phases with different physical properties. Therefore, single-phase flow models based only on Darcy's law are insufficient to describe phase interactions, saturation effects, and relative permeability behavior. Multiphase flow models extend the classical porous media formulation by considering phase-dependent properties, including viscosity, density, pressure distribution, and relative permeability. The relative permeability concept describes the ability of each fluid phase to flow through the porous structure when multiple phases occupy the pore space. It depends mainly on fluid saturation, pore geometry, and capillary effects. These models provide the theoretical foundation for describing oil–water displacement, filtration efficiency, and transient multiphase transport processes in heterogeneous porous systems.

Porosity and permeability are fundamental structural parameters controlling transport phenomena in porous media. Porosity defines the available pore volume, whereas permeability determines the ability of fluids to migrate through the porous structure. Classical models, such as the Kozeny–Carman relationship, establish correlations between permeability and structural characteristics of porous materials. However, these empirical approaches are often limited when applied to heterogeneous multilayer porous media due to variations in pore size distribution, connectivity, and tortuosity.

Recent studies demonstrate that machine learning approaches can overcome some limitations of conventional correlations by identifying nonlinear relationships between pore-scale structural features and effective transport properties. The transport of dispersed oil droplets, suspended particles, and contaminants during filtration is commonly described using convection–diffusion–deposition models. These models consider the combined influence of fluid convection, molecular or Brownian diffusion, and deposition mechanisms responsible for particle retention within porous structures. During filtration, deposited materials gradually modify the internal pore geometry, resulting in permeability reduction and changes in flow resistance. This interaction between transport processes and structural evolution introduces strong nonlinear behavior, making accurate prediction difficult using conventional analytical approaches.

In practical reservoir and filtration systems, pressure distribution, fluid velocity, saturation, and permeability may vary continuously with time. Therefore, oil filtration processes are considered nonstationary and require transient mathematical models. Multilayer porous systems introduce additional complexity due to differences in layer properties, including permeability, porosity, and fluid transport characteristics. These variations lead to nonuniform pressure fields and complex flow patterns that are challenging to capture using simplified homogeneous models. Consequently, advanced computational approaches, including numerical simulation and artificial intelligence-based methods, are increasingly applied to improve prediction accuracy and computational efficiency for nonstationary filtration problems.

Computational Methods for Oil Filtration Modeling

The mathematical description of oil filtration processes in heterogeneous porous media requires efficient numerical methods capable of solving complex transport equations under different spatial and temporal scales. Depending on the level of detail required, computational approaches can be classified into continuum-scale methods, pore-scale simulations, and image-based modeling techniques. These methods provide different levels of accuracy and computational efficiency and are often combined within multiscale modeling frameworks.

Computational Fluid Dynamics (CFD) is one of the most widely used numerical approaches for investigating fluid flow, mass transfer, and multiphase transport phenomena in oil filtration systems. CFD methods are based on the numerical solution of conservation equations, including the Navier–Stokes equations, continuity equations, and transport equations. For filtration systems involving porous structures, the Navier–Stokes equations are often coupled with Darcy or Brinkman formulations. The Brinkman equation extends Darcy's law by considering viscous diffusion effects and can be expressed as:

$$-\nabla P + \mu\nabla^2 u - \frac{\mu}{k}u = 0 \quad (1)$$

where:

P is pressure;

u is fluid velocity;

k is permeability;

μ is dynamic viscosity.

CFD modeling allows researchers to investigate:

- velocity distribution inside filtration devices;
- pressure drop across porous layers;
- oil droplet migration;
- concentration polarization;
- membrane fouling mechanisms.

Multiphase CFD approaches, such as the Euler–Euler model and Volume of Fluid (VOF) method, are frequently applied for oil-water separation processes. The VOF method is particularly useful for tracking oil-water interfaces and analyzing droplet deformation and coalescence phenomena. However, CFD simulations of oil filtration systems have several limitations. High-resolution simulations require significant computational resources, especially when pore-scale structures and transient multiphase interactions are considered. In addition, accurate CFD prediction depends on appropriate closure relationships for droplet behavior, turbulence effects, and deposition mechanisms.

Numerical discretization methods play a fundamental role in solving mathematical models of oil filtration processes. Among them, the finite difference method (FDM) and finite element method (FEM) are widely applied in reservoir simulation and porous media transport problems.

The finite difference method approximates differential equations by replacing derivatives with algebraic expressions. For example, the pressure gradient can be discretized as:

$$\frac{\partial P}{\partial x} \approx \frac{P_{i+1} - P_i}{\Delta x} \quad (2)$$

FDM is computationally efficient and suitable for regular computational domains. It has been widely used for solving transient filtration equations and pressure distribution problems. The finite element method provides greater flexibility for complex geometries and heterogeneous porous structures. In FEM, the computational domain is divided into small elements, and the governing equations are solved using approximation functions. Advantages of FEM include:

- ability to model irregular geometries;
- accurate representation of heterogeneous media;
- compatibility with coupled physical processes.

For multilayer porous media, FEM is particularly effective because different layers can be assigned individual properties such as permeability, porosity, and fluid characteristics.Despite these advantages, both FDM and FEM require accurate input parameters and may become computationally expensive for large-scale three-dimensional transient simulations.

Pore Network Modeling (PNM) is a pore-scale computational approach that represents porous materials as interconnected networks of pores and throats. Unlike continuum models, PNM explicitly considers the internal geometry of porous structures and provides detailed information about microscale transport mechanisms. In a pore network model, pores are represented as nodes, connections between pores are represented as throats, and fluid movement is calculated based on local pressure differences and capillary forces.

The flow rate between two connected pores can be estimated using the Hagen–Poiseuille relationship:

$$Q = \frac{\pi r^4}{8\mu L}\Delta P \quad (3)$$

where:

r is pore radius;

L is throat length;

ΔP is pressure difference.

PNM has been successfully applied for studying oil droplet capture, pore blockage, permeability changes, and multiphase displacement. The main advantage of pore network modeling is the ability to capture microscopic filtration mechanisms with lower computational cost compared with fully resolved CFD simulations. However, the accuracy of PNM strongly depends on the quality of the generated pore network. Simplified pore structures may not fully represent real geological formations with complex connectivity and heterogeneity.

Lattice Boltzmann Method (LBM)

The Lattice Boltzmann Method is an advanced numerical approach that describes fluid motion at the mesoscopic scale. Unlike conventional CFD methods, which directly solve macroscopic conservation equations, LBM simulates particle distribution functions on a discrete lattice.The evolution of particle distribution is described by:

$$f_i(x + c_i\Delta t, t + \Delta t) = f_i(x,t) - \frac{1}{\tau}(f_i - f_i^{eq}) \quad (4)$$

where:

f_i represents distribution functions;

c_i is lattice velocity;

τ is relaxation time.

LBM has several advantages for porous media modeling, including natural treatment of complex boundaries, efficient simulation of multiphase flow, and direct application to digital pore geometries. It has been widely used for permeability prediction, droplet transport, fluid-fluid interface simulation, and pore-scale filtration analysis. Nevertheless, LBM simulations remain computationally demanding when applied to large three-dimensional porous structures.

Digital Rock Physics (DRP) represents a modern approach that combines high-resolution imaging techniques with numerical simulation. In this approach, real porous structures are reconstructed from imaging data, such as X-ray micro-computed tomography (micro-CT). The general workflow includes: image acquisition; image processing and segmentation; three-dimensional pore reconstruction; numerical flow simulation; and extraction of transport parameters.

Digital rock models enable direct estimation of permeability, porosity, tortuosity, pore connectivity, and multiphase transport characteristics. The major advantage of DRP is that it eliminates the need for idealized pore assumptions and provides realistic representations of heterogeneous porous media. However, several challenges remain, including high computational requirements, dependence on imaging resolution, and difficulties in processing large datasets. Recently, artificial intelligence methods, particularly convolutional neural networks (CNNs), have been integrated into digital rock workflows to accelerate image segmentation, reconstruction, and property prediction.

Summary of Computational Approaches

Different computational methods provide complementary capabilities for oil filtration modeling, as summarized in Table 1.

Table 1

Summary of computational approaches for oil filtration modeling

Method	Modeling scale	Main advantages	Main limitations
Darcy-based models	Field scale	Fast and simple	Requires empirical parameters
CFD	Device to field scale	Detailed flow analysis	High computational cost
FEM/FDM	Continuum scale	Flexible numerical solution	Parameter dependent
Pore network models	Pore scale	Captures microscopic mechanisms	Simplified geometry
LBM	Pore scale	Complex multiphase simulation	Computationally intensive
Digital Rock Physics	Micro-to-core scale	Realistic structures	Requires imaging data

The development of these computational approaches has created a strong foundation for integrating artificial intelligence methods into oil filtration modeling. In particular, AI-based surrogate models and physics-informed neural networks provide new opportunities to overcome computational limitations and improve prediction accuracy for complex nonstationary filtration processes.

Artificial Intelligence Approaches for Oil Filtration Modeling

In recent years, artificial intelligence (AI) has become an important research direction for improving the prediction, optimization, and control of complex transport processes in porous media. Traditional mathematical models provide strong physical interpretations; however, their application to heterogeneous and nonstationary filtration systems is often limited by computational complexity and the requirement for accurately measured parameters. AI-based approaches offer alternative solutions by learning complex nonlinear relationships between input variables and filtration behavior.Artificial intelligence methods applied to oil filtration modeling can generally be classified into machine learning models, deep learning approaches, physics-informed neural networks, and hybrid physics–AI frameworks.

Machine learning (ML) techniques have been widely applied for predicting important porous media properties and filtration performance indicators. Unlike conventional numerical models, machine learning algorithms identify relationships directly from available experimental or simulation data.The general form of a machine learning prediction model can be expressed as:

$$Y=f(X;\theta) \tag{5}$$

where:

- X represents input parameters;
- Y represents predicted output;
- f is the learned nonlinear function;
- θ represents model parameters.

For oil filtration systems, typical input variables include porosity, permeability, pressure gradient, temperature, viscosity, saturation, and pore structure parameters. The predicted outputs may include filtration efficiency, pressure distribution, permeability variation, and oil-water separation performance.

Artificial Neural Networks (ANN)

ANN models are inspired by biological neural systems and consist of interconnected layers of neurons. They are capable of approximating highly nonlinear relationships between filtration parameters. ANNs have been successfully applied for permeability estimation, prediction of oil recovery parameters, and filtration efficiency forecasting. Their main advantage is the ability to model complex nonlinear dependencies without requiring explicit mathematical relationships. However, ANN models require sufficient training data and may produce physically unrealistic results when applied outside the training range.

Support Vector Machine (SVM)

Support Vector Machine algorithms are supervised learning methods that perform classification and regression by constructing optimal decision boundaries. For regression problems, Support Vector Regression (SVR) has been applied to permeability prediction, membrane performance estimation, and separation efficiency modeling. SVM models perform well with limited datasets but may become computationally expensive when the number of training samples increases.

Ensemble Learning Methods

Tree-based algorithms such as Random Forest, Gradient Boosting, and Extreme Gradient Boosting (XGBoost) have also been applied for porous media property prediction. These methods provide high prediction accuracy, resistance to noise, and estimation of variable importance. They are particularly useful when filtration datasets contain multiple interacting parameters.

Deep learning methods have significantly expanded the capabilities of AI-based filtration modeling by enabling automatic extraction of complex patterns from large datasets. Among deep learning methods, convolutional neural networks (CNNs) are especially important for digital rock physics applications. CNNs can process spatial information from micro-CT images, pore-scale simulations, and microscopic filter structures

$$\mathcal{Image} \rightarrow \mathcal{Feature} \rightarrow \mathcal{Prediction}$$

CNN-based models have been applied for pore structure recognition, permeability prediction, porosity estimation, and segmentation of rock and filter images. Compared with traditional image-processing techniques, CNNs reduce the need for manual feature extraction and can identify hidden structural characteristics influencing fluid transport.

However, deep learning models require large datasets and their physical interpretability remains limited.

Physics-Informed Neural Networks represent one of the most promising AI approaches for modeling complex filtration processes. Unlike conventional machine learning models, PINNs incorporate governing physical equations directly into the neural network training process.The main concept of PINN is to minimize a combined loss function:

$$L_{oss}=L_{data}+L_{physics} \tag{6}$$

where:

- L_{data} represents the difference between predicted and observed data;
- $L_{physics}$ represents the violation of governing physical equations.

For oil filtration problems, the physical component may include:

Darcy's law:

$$u = -\frac{k}{\mu}\nabla p \tag{7}$$

Mass conservation equation:

$$\nabla \cdot u = 0 \tag{8}$$

By embedding these equations into the learning process, PINNs can generate solutions that satisfy fundamental physical principles.

Nonstationary filtration in multilayer porous media involves time-dependent pressure distribution, variable permeability, and complex boundary conditions. These characteristics make conventional numerical simulations computationally expensive. PINNs provide several advantages, including reduced requirement for simulation data, the ability to solve inverse problems, estimation of unknown parameters, and faster prediction after training.A possible PINN-based framework for multilayer oil filtration can be represented as:

Physical parameters (permeability, porosity, viscosity)
↓
Governing equations (Darcy + continuity + transport)
↓
Physics-informed neural network
↓
Pressure and velocity prediction
↓
Filtration optimization

Such models can be used for prediction of pressure fields, estimation of permeability distribution, identification of unknown reservoir parameters, and optimization of filtration processes.

Hybrid Physics–Artificial Intelligence Models

Although AI models demonstrate high prediction capability, purely data-driven approaches may violate physical constraints. Therefore, hybrid physics–AI models are considered a promising direction for future oil filtration research. A hybrid framework combines physical equations, numerical simulation, and machine learning algorithms, following the general structure:

Physical Model + AI Model → Improved Prediction

Examples include CFD-generated datasets used for neural network training, PINNs accelerating numerical simulation, AI-based correction of empirical parameters, and surrogate models replacing expensive simulations. For multilayer porous media, hybrid models can combine Darcy-based flow equations, heterogeneous permeability models, neural networks, and experimental observations. This approach provides a balance between physical reliability and computational efficiency.

Challenges of AI Application in Oil Filtration Modeling

Despite significant progress, several challenges remain:

Data limitations

Oil filtration datasets are often limited because experimental measurements under reservoir conditions are expensive.

Model generalization

AI models trained under specific conditions may not perform well when applied to different rock types, fluid compositions, or operating conditions.

Interpretability

Deep learning models often operate as black boxes, making physical interpretation difficult.

Integration with numerical models

Developing stable hybrid AI–physics frameworks remain an active research area.

Future Perspective of AI-Based Oil Filtration Modeling

The future development of oil filtration modeling is expected to move toward intelligent hybrid frameworks combining mathematical models, high-performance computing, artificial intelligence, and real-time monitoring systems. The integration of PINNs, digital twins, and advanced numerical simulations may enable accurate prediction and control of nonstationary filtration processes in heterogeneous multilayer porous media.

3. Results and Discussion

The development of oil filtration modeling has progressed from simplified analytical approaches toward advanced computational and artificial intelligence-based frameworks. Each modeling approach provides specific advantages depending on the required accuracy, computational The development of oil filtration modeling has progressed from simplified analytical approaches toward advanced computational and artificial intelligence-based frameworks. Each modeling approach provides specific advantages depending on the required accuracy, computational resources, and available data. Classical mathematical models remain essential because they provide a clear physical interpretation of transport mechanisms, while artificial intelligence methods offer improved prediction capabilities for complex nonlinear systems. A detailed comparison of the conventional computational methods discussed in Section 3 is provided in Table 1 above.

Artificial intelligence methods have introduced new possibilities for predicting filtration behavior by learning complex relationships between input parameters and system responses. Different AI techniques demonstrate different capabilities depending on the available data type and modeling objective, as summarized in Table 2.

Table 2

Comparison of AI methods applied to oil filtration modeling

AI Method	Input data	Predicted parameters	Advantages	Limitations
Artificial Neural Network (ANN)	Experimental and simulation data	Permeability, pressure, filtration efficiency	Effective for nonlinear relationships	Requires sufficient training data
Support Vector Regression (SVR)	Process parameters	Separation efficiency, transport properties	Performs well with small datasets	Limited scalability
Random Forest	Multivariable datasets	Reservoir and filtration parameters	Robust and interpretable	Lower performance for complex spatial problems
Convolutional Neural Network (CNN)	Pore images and CT data	Porosity, permeability, pore structure	Automatic feature extraction	Requires large image datasets
Physics-Informed Neural Network (PINN)	Physical equations + data	Pressure and velocity fields	Maintains physical consistency	Training complexity

Traditional mathematical models and AI approaches should not be considered competing methods; rather, they provide complementary capabilities. Classical models such as Darcy-based formulations have several important advantages, including strong physical interpretation, conservation law satisfaction, and applicability with limited data. However, they face difficulties in representing heterogeneous structures, predicting nonlinear behavior, and performing rapid optimization.

AI methods provide fast prediction after training, the ability to capture nonlinear relationships, and efficient processing of large datasets. Nevertheless, purely data-driven models may suffer from a lack of physical constraints, poor extrapolation capability, and dependence on training data quality. Therefore, hybrid approaches combining physical models and artificial intelligence represent the most promising direction for future oil filtration modeling.

For multilayer porous systems, the main challenge is the simultaneous consideration of spatially varying permeability, pressure-dependent properties, multiphase transport, and transient boundary conditions. Conventional numerical methods can accurately solve such problems but often require significant computational resources.

PINNs provide an alternative approach by incorporating governing equations directly into the neural network training process. For multilayer filtration systems, the model can be formulated as:

$$\frac{\partial P}{\partial t} = \nabla\left(k\frac{(x,y,z,P)}{\mu}\nabla P\right) \tag{9}$$

where the unknown pressure field $P(x,y,z,t)$ is approximated by a neural network:

$$P = N(x, y, z, t; \theta) \tag{10}$$

The network parameters θ are optimized by minimizing the physics-based and data-based losses. This approach enables reconstruction of pressure fields, estimation of unknown permeability distributions, reduction of simulation time, and integration of experimental measurements. For nonstationary oil filtration problems, PINNs can serve as surrogate models replacing repeated computational simulations while preserving physical consistency.

Research Gaps and Future Challenges

Despite significant progress, several research challenges remain unresolved.

1. Limited experimental data

AI models require representative datasets. However, oil filtration experiments under realistic reservoir conditions are expensive and time-consuming. Future research should focus on standardized datasets, integration of experimental and numerical data, and transfer learning approaches.

2. Multiscale modeling challenges

Oil filtration occurs across different spatial scales — pore scale, core scale, and reservoir scale. Connecting these scales remains difficult because information loss may occur during model transitions. Future directions include AI-assisted multiscale models, digital twin technologies, and hierarchical neural architectures.

3. Model interpretability

Although deep learning models demonstrate high accuracy, their internal decision-making processes remain difficult to interpret. The development of explainable AI (XAI), physics-guided neural networks, and interpretable machine learning is necessary for industrial implementation.

4. Real-time prediction and optimization

Future intelligent filtration systems should provide real-time monitoring, automatic parameter adjustment, and predictive maintenance. The integration of AI models with sensor technologies and high-performance computing may enable the development of intelligent filtration control systems.

Proposed Framework for AI-Based Nonstationary Oil Filtration Modeling

Based on the reviewed approaches, a hybrid computational framework for multilayer porous media can be proposed:

Multilayer porous media properties (permeability, porosity, viscosity)

↓

Mathematical model (Darcy equation + transport equations)

↓

Numerical simulation (FDM / FEM / CFD)

↓

Physics-informed AI model (PINN + machine learning)

↓

Pressure and filtration prediction

↓

Optimization and digital twin system

This framework combines the reliability of physical modeling with the computational efficiency of artificial intelligence. It provides a promising basis for developing advanced predictive models for nonstationary oil filtration processes.

4. Conclusion

Oil filtration processes in porous media represent complex physical systems characterized by multiphase transport, heterogeneous structures, and time-dependent changes in flow properties. Accurate modeling of these processes is essential for improving oil-water separation efficiency, optimizing reservoir management, and developing advanced filtration technologies.This review analyzed the main mathematical, computational, and artificial intelligence approaches applied to oil filtration modeling in multilayer porous media. Classical mathematical models based on Darcy's law, conservation equations, and transport models remain fundamental tools for describing fluid movement and filtration mechanisms. These approaches provide strong physical interpretation and reliability; however, their application to highly heterogeneous and nonlinear systems is often limited by computational complexity and the requirement for experimentally determined parameters.Advanced computational methods, including computational fluid dynamics, finite difference and finite element methods, pore network modeling, lattice Boltzmann simulations, and digital rock physics, have significantly improved the ability to investigate filtration phenomena across different spatial scales. These methods provide detailed information about pore-scale transport, multiphase interactions, and structural evolution during filtration processes. Nevertheless, their high computational cost remains a major challenge, especially for large-scale transient simulations and optimization problems.

Artificial intelligence methods have recently demonstrated significant potential for improving prediction and analysis of porous media filtration processes. Machine learning and deep learning algorithms can effectively identify nonlinear relationships between system parameters and filtration performance indicators. In particular, convolutional neural networks have enhanced digital rock analysis, while artificial neural networks have been successfully applied for permeability and transport property prediction.Among AI-based approaches, physics-informed neural networks represent one of the most promising directions because they combine the advantages of data-driven models with the physical consistency of mathematical equations. By incorporating governing equations such as Darcy's law and mass conservation principles into the learning process, PINNs provide opportunities for developing accurate and computationally efficient models for nonstationary filtration processes in heterogeneous multilayer porous media.

However, several challenges remain before AI-based filtration models can be widely implemented in practical engineering applications. These challenges include limited availability of high-quality experimental datasets, difficulties in model generalization across different geological conditions, uncertainty quantification, and limited interpretability of deep learning models. Addressing these issues requires further development of hybrid physics–AI approaches, explainable artificial intelligence methods, and multiscale computational frameworks.

Future research should focus on integrating mathematical models, numerical simulation techniques, artificial intelligence algorithms, and real-time monitoring technologies into unified intelligent filtration systems. Such hybrid frameworks have the potential to provide accurate prediction, optimization, and control of nonstationary oil filtration processes in complex multilayer porous media.

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